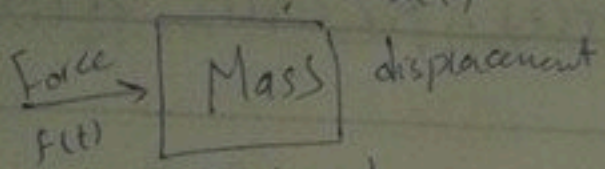
Mechanical System

Control

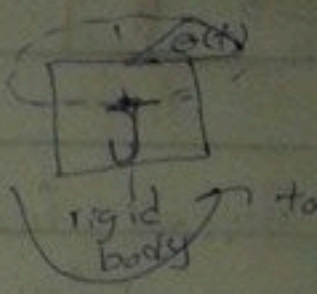
Linear (translational)



$W = Mg$

$\dot{x}(t)$

Rotational



$\theta(t)$ = angular displacement

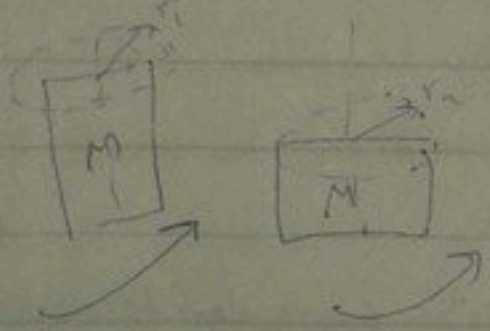
J = moment of inertia

$$J = \frac{1}{2} \mu r^2$$

Resistance to motion

$$\dot{\theta}(t) = \frac{d\theta(t)}{dt} = \omega$$

$$\frac{d^2\theta}{dt} = \frac{d\omega}{dt} = \alpha$$ angular acceleration

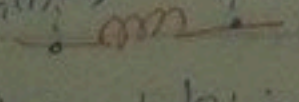


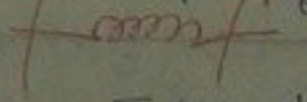
inertia

$$J_1 = \frac{1}{2} M r_1^2$$

$$J_2 = \frac{1}{2} M r_2^2$$

② spring

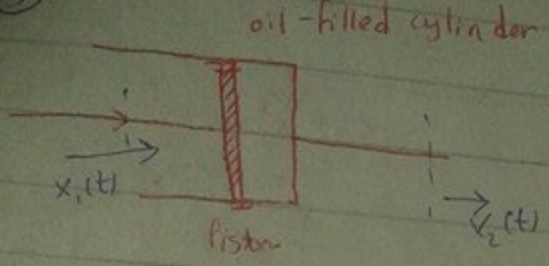


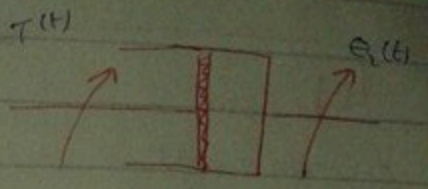


$$T(t) = K[\theta_1(t) - \theta_2(t)]$$

$$f(t) = K[x_1(t) - x_2(t)]$$

(3) damper = dashpot



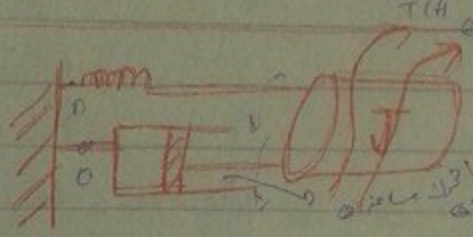


$T(t) = \beta[\dot{\theta}_1(t) - \dot{\theta}_2(t)$

Viscous friction coefficient β

$f(t) = \beta[\dot{x}_1(t) - \dot{x}_2(t)]$

Rotational mechanical systems



① write the differential eqn

② Find the transfer fn.

Newton second law of motion

linear (translational)

mass * acceleration

= Σ forces on the mass

input = force $f(t)$ (+)

output = displacement $z(t)$

rotational

moment of inertia * angular acceleration

= Σ torques on the inertia

$J \cdot \ddot{\theta}(t) = T(t) - \beta[\dot{\theta}(t) - 0] - K[\theta(t) - 0]$

$T(t)$ → input

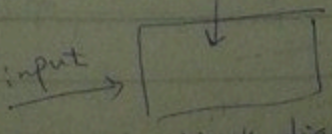
$\theta(t)$ → out

→ force of spring & damper = $(-vc)$

→ $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = T(t)$

acceleration

$\alpha = \ddot{\theta}(t)$



block diagram

system ν linear

① $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = T(t)$

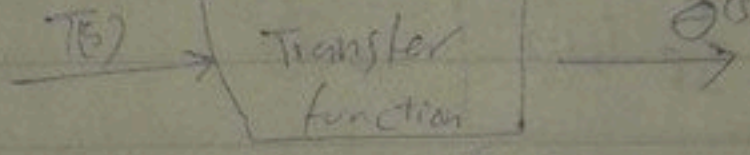
② Transfer fn $G(s) = \frac{L(o/p)}{L(i/p)} = \frac{\theta(s)}{T(s)} = ??$

laplace transforme of differential equations.

$$J s^2 \theta(s) + B s \theta(s) + K\theta(s) = T(s)$$

$$\therefore \theta(s)[J s^2 + Bs + K] = T(s)$$

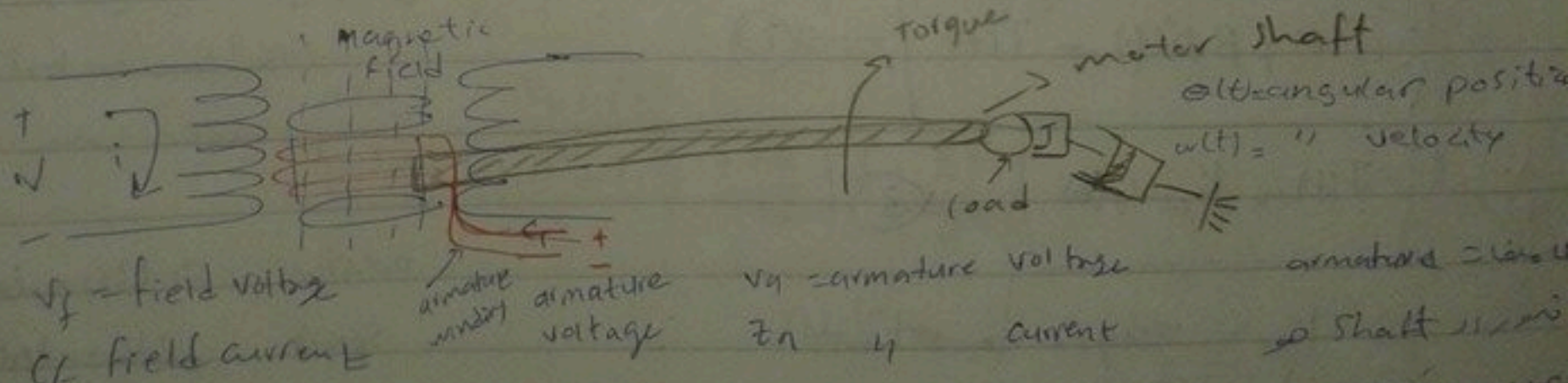
$$\frac{\theta(s)}{T(s)} = \frac{1/J}{s^2 + \frac{B}{J}s + K/J}$$



Field contrd DC moto

دستگاه حلقه باز

③ Electromechanical system ⟶ DC motor → armature control motor



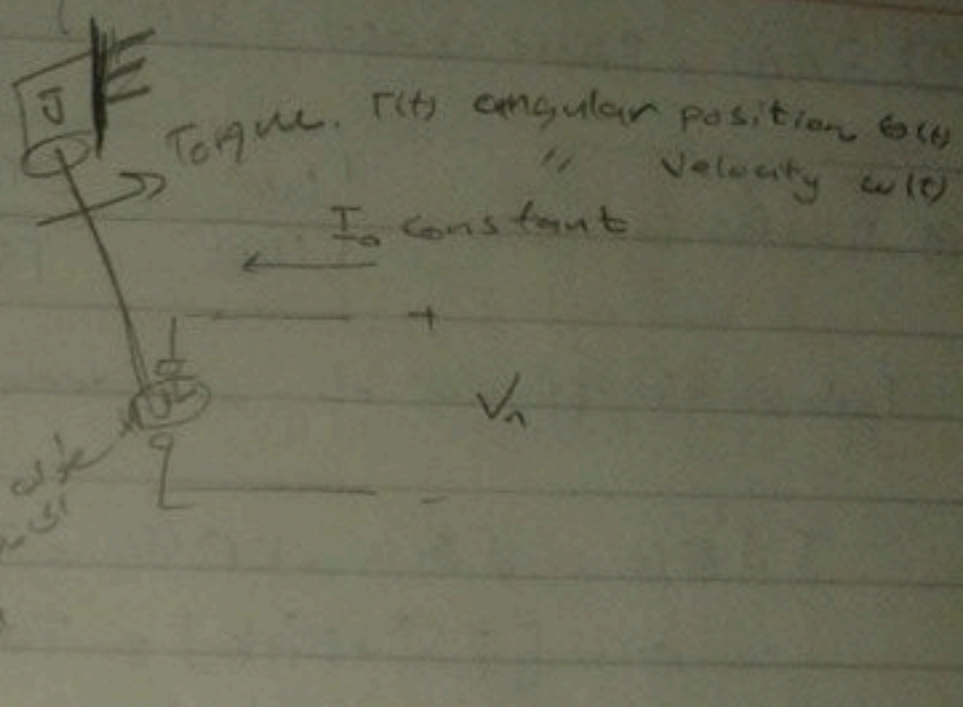
V_f = field voltage
i_f field current
armature voltage
V_a = armature voltage
i_a " current
armature = آرمیچر
Shaft = شفت

field Control
کنترل میدان

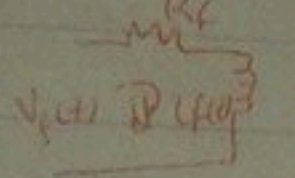
armature Control
کنترل آرمیچر

Field control DC motor

friction damper



equivlant circuit



R_f = Field resistance

L_f = Field inductance

& mathematical modeling

KVL around field circuit

$$V_f(t) = R_f i_f(t) + L_f \frac{d}{dt} i_f(t) \longrightarrow (1)$$

العلاقة بين الجزء الكهربي والجزء الميكانيكي.

$$J\ddot{\theta}(t) = T(t) - \beta\dot{\theta}(t)$$

$$J\ddot{\theta}(t) + \beta\dot{\theta}(t) = T(t) \longrightarrow (2)$$

$$T(t) = K_t\, i_f(t) \longrightarrow (3)$$

K_t = torque constant

$$\omega(t) = \frac{d}{dt}\theta(t) \longrightarrow (4)$$

الكونترولر يمكن التحكم في السرعة بالكنترول او الزاوية بالكنترول اعتماد على

position control

speed control

position $\theta(t)$ = output

Speed $\omega(t)$ output

(5) in (1)

$$\omega = \frac{d\theta}{dt} \quad (5)$$

$$V_f(t) = \frac{R_f}{K_t} T(t) + \frac{L_f}{K_t} \frac{d}{dt} T(t) \longrightarrow (5)$$

$$V_f(t) = \frac{R_f}{K_t} \left[J\ddot{\theta}(t) + B\dot{\theta}(t) \right] + \frac{L_f}{K_t} \left[J\dddot{\theta}(t) + B\ddot{\theta}(t) \right] \longrightarrow (6)$$

(2) in (5)

$$V_f(t) = \frac{JL_f}{K_t} \dddot{\theta}(t) + \left[\frac{BL_f}{K_t} + \frac{JR_f}{K_t} \right] \ddot{\theta}(t) + \frac{BR_f}{K_t} \dot{\theta}(t)$$

$$\dddot{\theta}(t) + \left[\frac{B}{J} + \frac{R_f}{L_f} \right] \ddot{\theta}(t) + \frac{BR_f}{JL_f} \dot{\theta}(t) = \frac{K_t}{JL_f} V_f(t)$$

if $\omega(t)$ = output

$$\ddot{\omega}(t) + \left[\frac{B}{J} + \frac{R_f}{L_f} \right] \dot{\omega}(t) + \frac{BR_f}{JL_f} \omega(t) = \frac{K_t}{JL_f} V_f(t)$$

بناء على الدخل والخرج، هنا حصلنا على نظام مغلق.

$$G(s) = \frac{\Theta(s)}{V_f(s)} = \frac{K_t / JL_f}{s^3 + \left[\frac{B}{J} + \frac{R_f}{L_f} \right] s^2 + \frac{BR_f}{JL_f} s}$$

نأخذ Θ output

$$G(s) = \frac{\omega(s)}{V_f(s)} = \frac{K_t / JL_f}{s^2 + \left[\frac{B}{J} + \frac{R_f}{L_f} \right] s + \frac{BR_f}{JL_f}}$$

يمكن إيجاد الـ Transfer function بطريقة ثانية

(4) بأخذ laplace قبل ما نجمع

$$V_f(s) = \left[R_f + sL_f \right] I_f(s) \longrightarrow (1)'$$

$$\left[Js^2 + Bs \right] \Theta(s) + T(s) \longrightarrow (2)'$$

$$T(s) = K_t I_f(s) \longrightarrow (3)'$$

$$\omega(s) = s\theta(s) \longrightarrow (4)'$$